

Production of photons by the parametric resonance in the dynamical Casimir effect

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Abstract

We calculate the number of photons produced by the parametric resonance in a cavity with vibrating walls. We consider the case that the frequency of vibrating wall is $n\omega_1$ ($n = 1, 2, 3, \dots$) which is a generalization of other works considering only $2\omega_1$, where ω_1 is the fundamental-mode frequency of the electromagnetic field in the cavity. For the calculation of time-evolution of quantum fields, we introduce a new method which is borrowed from the time-dependent perturbation theory of the usual quantum mechanics. This perturbation method makes it possible to calculate the photon number for any n and to observe clearly the effect of the parametric resonance.

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I. INTRODUCTION

Recently, the photon creation in an empty cavity with moving boundaries, so called the dynamical Casimir effect [1] has attracted much attention especially in a vibrating cavity [2–5]. It was also proposed that the high- Q electromagnetic cavities may provide a possibility to detect the photons produced in the dynamical Casimir effect [7,8]. Therein, they considered the vibrating wall with the frequency $\Omega = 2\omega_1$ and found the resonance excitation of the electromagnetic modes.

For the solutions of Mathieu differential equation

$$\ddot{x} + \mu^2(1 + \epsilon \cos \Omega t)x = 0, \quad (1.1)$$

it is well known that parametric resonances occur when the frequency Ω with which the parameter oscillates is close to any value $2\mu/n$ with n integral [9]. Since there are many mode frequencies for the differential equation describing the electromagnetic fields in a cavity, it is natural to consider other frequencies with $\Omega = n\omega_1$. Now the method introduced in Ref. [8] cannot be used in these general cases because the generating function can be made only for the special case $n = 2$. Therefore we introduce a new perturbation method which leads to easily solvable equations. In principle, we may solve the equations to any order of expansion parameter ϵ , but we calculate up to the first order of ϵ , which is sufficient to see the effect of the parametric resonance.

The organization of this paper is as follows. In Sec. II we review the scheme of the field quantization in the case of moving boundaries. In Sec. III we introduce the new perturbation method to find time evolution of quantum electromagnetic field. Here we write the dominant part of the solution of wave equation which results from the parametric resonance. In Sec. IV we calculate the number of photons created by the vibration of the boundary. The last section is devoted to the summary and discussion. Here, we discuss the physical properties of the parametric resonance in the coupled differential equations, and we estimate the photon number created in realistic situation. Finally, the higher order calculations are considered briefly.

II. QUANTUM ELECTROMAGNETIC FIELDS IN A CAVITY WITH MOVING WALLS

Let us consider an empty cavity formed by two perfect conducting walls, one being at rest at $x = 0$ and the other moving according to a given law of motion $L(t)$ when $0 < t < T$. The field operator in the Heisenberg representation $A(x, t)$ associated with a vector potential obeys the wave equation ($c = 1$)

$$\frac{\partial^2 A}{\partial t^2} - \frac{\partial^2 A}{\partial x^2} = 0 \quad (2.1)$$

and can be written as

$$A(x, t) = \sum_n [b_n \psi_n(x, t) + b_n^\dagger \psi_n^*(x, t)]. \quad (2.2)$$

Here b_n^\dagger and b_n are the creation and the annihilation operators and $\psi_n(x, t)$ is the corresponding mode function which satisfies the boundary condition $\psi_n(0, t) = 0 = \psi_n(L(t), t)$.

For an arbitrary moment of time, following the approach of Refs. [10–12], we expand the mode function as

$$\psi_n(x, t) = \sum_k Q_{nk}(t) \varphi_k(x, t) \quad (2.3)$$

with the instantaneous basis

$$\varphi_k(x, L(t)) = \sqrt{\frac{2}{L(t)}} \sin \frac{\pi k x}{L(t)}. \quad (2.4)$$

Here $Q_{nk}(t)$ obeys an infinite set of coupled differential equations [6]:

$$\begin{aligned} \ddot{Q}_{nk} + \omega_k^2(t) Q_{nk} &= 2\lambda \sum_j g_{kj} \dot{Q}_{nj} + \dot{\lambda} \sum_j g_{kj} Q_{nj} \\ &\quad + \dot{\lambda}^2 \sum_{j,l} g_{jk} g_{jl} Q_{nl} \end{aligned} \quad (2.5)$$

where $\lambda = \dot{L}/L$ and

$$g_{kj} = \begin{cases} (-1)^{k-j} \frac{2kj}{j^2 - k^2} & (j \neq k) \\ 0 & (j = k) \end{cases}. \quad (2.6)$$

and the time-dependent mode frequency is

$$\omega_k(t) = \frac{k\pi}{L(t)}. \quad (2.7)$$

For $t \leq 0$, the right hand side of Eq. (2.5) vanishes and the solution in this region is chosen to be

$$Q_{nk}(t) = \frac{e^{-i\omega_k t}}{\sqrt{2\omega_k}} \delta_{nk}. \quad (2.8)$$

Then the quantum field can be written as

$$A(x, t \leq 0) = \sum_n [b_n \frac{e^{-i\omega_n t}}{\sqrt{2\omega_n}} \varphi_n(x, L_0) + \text{H.c.}] \quad (2.9)$$

L_0 is the initial distance between the walls and $\omega_n = \frac{\pi n}{L_0}$. From the form of the Hamiltonian $H = \sum_n \omega_n (b_n^\dagger b_n + \frac{1}{2})$, we can interpret $b_n^\dagger b_n$ as the number operator associated with the photon with the frequency ω_n .

After the oscillation of the wall ($t \geq T$), the solution of Eq. (2.5) with the initial condition (2.8), can be written as

$$Q_{nk}(t \geq T) = \alpha_{nk} \frac{e^{-i\omega_k t}}{\sqrt{2\omega_k}} + \beta_{nk} \frac{e^{i\omega_k t}}{\sqrt{2\omega_k}}. \quad (2.10)$$

From (2.2) and (2.3), we have

$$A(x, t \geq T) = \sum_n [a_n \frac{e^{-i\omega_n t}}{\sqrt{2\omega_n}} \varphi_n(x, L_0) + \text{H.c.}], \quad (2.11)$$

where the new creation and annihilation operators a_n^\dagger and a_n are written in terms of b_n^\dagger and b_n , using the following Bogoliubov transformations

$$\begin{aligned} a_k &= \sum_n [b_n \alpha_{nk} + b_n^\dagger \beta_{nk}^*] \\ a_k^\dagger &= \sum_n [b_n^\dagger \alpha_{nk}^* + b_n \beta_{nk}]. \end{aligned} \quad (2.12)$$

Further, it follows from $H = \sum_n \omega_n (a_n^\dagger a_n + \frac{1}{2})$ that $a_n^\dagger a_n$ is the new number operator at $t \geq T$.

If we start with a vacuum state $|0_b\rangle$ such that $b|0_b\rangle = 0$, the expectation value of the new number operator is

$$N_k = \langle 0_b | a_k^\dagger a_k | 0_b \rangle = \sum_{n=1}^{\infty} |\beta_{nk}|^2, \quad (2.13)$$

which can be interpreted as the number of created photons. (One should note that the quantum state does not evolve in time in the Heisenberg picture.)

III. TIME EVOLUTION OF QUANTUM ELECTROMAGNETIC FIELDS IN A CAVITY WITH AN OSCILLATING WALL

In this section we find the time evolution of quantum field operator (2.2) by solving Eq. (2.5) with the motion of the wall given by

$$L(t) = L_0[1 + \epsilon \sin(\Omega t)]. \quad (3.1)$$

Here $\Omega = \gamma\omega_1 = \gamma\pi/L_0$ and ϵ is a small parameter characterized by the displacement of the wall. This is a generalization of the previous work in Ref. [8] where the special case $\gamma = 2$ was treated. For $\epsilon \ll 1$, having in mind that $\lambda(t) \sim \epsilon$ and taking the first order of ϵ in the mode frequency (2.7)

$$\omega_k(t) = \frac{k\pi}{L_0}[1 + \epsilon \sin(\Omega t)]^{-1}, \quad (3.2)$$

we can replace Eq. (2.5) by a pair of coupled first-order differential equations

$$\begin{aligned} \dot{Q}_{nk} &= P_{nk} \\ \dot{P}_{nk} &= -\omega_k^2(1 - 2\epsilon \sin \Omega t)Q_{nk} + 2\frac{\dot{L}}{L} \sum_j g_{kj} P_{nj} \\ &\quad + \frac{\ddot{L}}{L} \sum_j g_{kj} Q_{nj} + O(\epsilon^2). \end{aligned} \quad (3.3)$$

Let us now introduce

$$X_{n,k\mp} = \sqrt{\frac{\omega_k}{2}} \left(Q_{nk} \pm i \frac{P_{nk}}{\omega_k} \right) \quad (3.4)$$

or inversely,

$$\begin{aligned}
Q_{nk} &= \sqrt{\frac{1}{2\omega_k}}[X_{n,k-} + X_{n,k+}] \\
P_{nk} &= i\sqrt{\frac{\omega_k}{2}}[-X_{n,k-} + X_{n,k+}].
\end{aligned} \tag{3.5}$$

Then (3.3) reads

$$\begin{aligned}
\dot{X}_{n,k\mp} &= \mp i\omega_k X_{n,k-} \pm i\omega_k \epsilon \sin \Omega t [X_{n,k-} + X_{n,k+}] \\
&\mp \epsilon \Omega \cos \Omega t \sum_j g_{kj} \sqrt{\frac{\omega_j}{\omega_k}} [-X_{n,j-} + X_{n,j+}] \\
&\mp \frac{i}{2} \epsilon \Omega^2 \sin \Omega t \sum_j g_{kj} \frac{1}{\sqrt{\omega_j \omega_k}} [X_{n,j-} + X_{n,j+}].
\end{aligned} \tag{3.6}$$

Introducing the infinite dimensional column vector

$$\vec{X}_n(t) = \begin{pmatrix} X_{n,1-} \\ X_{n,1+} \\ X_{n,2-} \\ \vdots \end{pmatrix}, \tag{3.7}$$

the above equation can be written as a matrix form

$$\frac{d}{dt} \vec{X}_n(t) = V^{(0)} \vec{X}_n(t) + \epsilon V^{(1)} \vec{X}_n(t) \tag{3.8}$$

and $V^{(0)}$ and $V^{(1)}$ are matrices. The components of the matrices are

$$V_{k\sigma,j\sigma'}^{(0)} = i\omega_k \sigma \delta_{kj} \delta_{\sigma\sigma'} \tag{3.9}$$

and

$$V_{k\sigma,j\sigma'}^{(1)} = \sum_{s=\pm} \omega_1 v_{k\sigma,j\sigma'}^s e^{si\gamma\omega_1 t}, \tag{3.10}$$

where

$$v_{k\sigma,j\sigma'}^s = \sigma \gamma g_{kj} \sqrt{\frac{j}{k}} \left(\frac{\sigma'}{2} + s \frac{\gamma}{4j} \right) - s \sigma \frac{k}{2} \delta_{kj} \tag{3.11}$$

with $s, \sigma, \sigma' = +, -$. Here we used $\Omega = \gamma\omega_1$ and $\omega_k = k\omega_1$.

To find the solution of Eq. (3.8), we introduce a perturbation expansion:

$$\vec{X}_n = \vec{X}_n^{(0)} + \epsilon \vec{X}_n^{(1)} + \epsilon^2 \vec{X}_n^{(2)} + \dots \quad (3.12)$$

The iteration method used to solve this problem is similar to what we did in time-dependent perturbation theory. By inserting (3.12) into Eq. (3.8), identifying powers of ϵ yields a series of equations:

$$\frac{d}{dt} \vec{X}_n^{(0)} = V^{(0)} \vec{X}_n^{(0)} \quad (3.13)$$

$$\frac{d}{dt} \vec{X}_n^{(1)} = V^{(1)} \vec{X}_n^{(0)} + V^{(0)} \vec{X}_n^{(1)}. \quad (3.14)$$

From the initial condition (2.8), we have the solution to zeroth order equation (3.13)

$$X_{n,k\sigma}^{(0)} = \delta_{nk} \delta_{\sigma-} e^{-i\omega_k t}. \quad (3.15)$$

Further, the Eq. (3.14) is easily solved

$$X_{n,k\sigma}^{(1)}(t) = e^{\sigma i \omega_k t} \int_0^t dt' e^{-\sigma i \omega_k t'} \sum_{j,\sigma'} V_{k\sigma,j\sigma'}^{(1)} X_{n,j\sigma'}^{(0)}. \quad (3.16)$$

Using (3.15) and (3.10), it can be written explicitly as

$$\begin{aligned} X_{n,k\sigma}^{(1)}(t) &= \omega_1 e^{\sigma i k \omega_1 t} \int_0^t dt' e^{-\sigma i k \omega_1 t'} \sum_{j,\sigma'} (v_{k\sigma,j\sigma'}^- e^{-i\gamma \omega_1 t'} \\ &\quad + v_{k\sigma,j\sigma'}^+ e^{i\gamma \omega_1 t'}) \delta_{nj} \delta_{-\sigma'} e^{-i j \omega_1 t'} \\ &= \omega_1 e^{\sigma i k \omega_1 t} \int_0^t dt' (v_{k\sigma,n-}^- e^{-i(\sigma k + \gamma + n) \omega_1 t'} \\ &\quad + v_{k\sigma,n-}^+ e^{+i(-\sigma k + \gamma - n) \omega_1 t'}), \end{aligned} \quad (3.17)$$

that is,

$$\begin{aligned} X_{n,k+}^{(1)}(t) &= -v_{k-,n-}^- E_{n+\gamma+k}^{-k}(t) - v_{k-,n-}^+ E_{n-\gamma+k}^{-k}(t), \\ X_{n,k-}^{(1)}(t) &= v_{k-,n-}^- E_{n+\gamma-k}^k(t) + v_{k-,n-}^+ E_{n-\gamma-k}^k(t), \end{aligned} \quad (3.18)$$

where

$$E_m^k(t) = \begin{cases} \omega_1 t e^{-ik\omega_1 t}, & \text{for } m = 0, \\ \frac{i}{m} (e^{-i(m+k)\omega_1 t} - e^{-ik\omega_1 t}), & \text{for } m \neq 0. \end{cases} \quad (3.19)$$

Therefore we have, using (3.18) and (3.5),

$$\begin{aligned}
Q_{nk}(t) &= \frac{1}{\sqrt{2\omega_k}} e^{-i\omega_k t} \delta_{nk} \\
&+ \frac{\epsilon}{\sqrt{2\omega_k}} [-v_{k-,n-}^- E_{n+\gamma+k}^{-k}(t) - v_{k-,n-}^+ E_{n-\gamma+k}^{-k}(t) \\
&+ v_{k-,n-}^- E_{n+\gamma-k}^k(t) + v_{k-,n-}^+ E_{n-\gamma-k}^k(t)] \\
&+ O(\epsilon^2).
\end{aligned} \tag{3.20}$$

One should note that $Q_{nk}^{(1)}$ includes terms proportional to $\omega_1 t$ which are the effects of parametric resonance. In the usual situation, since $\omega_1 t \gg 1$, only the resonance terms are dominant and the solution (3.20) becomes by retaining only them:

$$\begin{aligned}
Q_{nk}(t) &\approx \frac{1}{\sqrt{2\omega_k}} e^{-i\omega_k t} \delta_{nk} \\
&+ \frac{\epsilon\omega_1 t}{\sqrt{2\omega_k}} [-v_{k-,n-}^+ e^{i\omega_k t} \delta_{k,\gamma-n} \\
&+ v_{k-,n-}^- e^{-i\omega_k t} \delta_{k,n+\gamma} + v_{k-,n-}^+ e^{-i\omega_k t} \delta_{k,n-\gamma}].
\end{aligned} \tag{3.21}$$

IV. NUMBER OF PHOTONS CREATED BY THE PARAMETRIC RESONANCE

After some time interval T the wall stops at $x = L_0$. Then the wave function becomes

$$\psi_n(x, t > T) = \sum_k \left[\alpha_{nk} \frac{e^{-i\omega_k t}}{\sqrt{2\omega_k}} + \beta_{nk} \frac{e^{i\omega_k t}}{\sqrt{2\omega_k}} \right] \varphi_k(x). \tag{4.1}$$

The mode function (2.3) and its time-derivative should be continuous at $t = T$:

$$\begin{aligned}
\sum_k Q_{nk}(T) \varphi_k &= \sum_k \left(\alpha_{nk} \frac{e^{-i\omega_k T}}{\sqrt{2\omega_k}} + \beta_{nk} \frac{e^{i\omega_k T}}{\sqrt{2\omega_k}} \right) \varphi_k \\
\sum_k \left(\dot{Q}_{nk}(T) \varphi_k + Q_{nk}(T) \dot{\varphi}_k \right) &= \\
\sum_k \left(-i\omega_k \alpha_{nk} \frac{e^{-i\omega_k T}}{\sqrt{2\omega_k}} + i\omega_k \beta_{nk} \frac{e^{i\omega_k T}}{\sqrt{2\omega_k}} \right) \varphi_k,
\end{aligned} \tag{4.2}$$

where we used (2.10) for the mode function at $t > T$. By multiplying φ_l to both sides of the above equations and integrating, we can get α_{nk} and β_{nk} , which are

$$\begin{aligned}
\alpha_{nk} &= \left(i\omega_k Q_{nk} - \dot{Q}_{nk} + \frac{\dot{L}}{L} \sum_l g_{kl} Q_{nl} \right) \frac{e^{i\omega_k T}}{i\sqrt{2\omega_k}} \\
\beta_{nk} &= \left(i\omega_k Q_{nk} + \dot{Q}_{nk} - \frac{\dot{L}}{L} \sum_l g_{kl} Q_{nl} \right) \frac{e^{-i\omega_k T}}{i\sqrt{2\omega_k}}.
\end{aligned} \tag{4.3}$$

Retaining only the dominant terms ($\omega_1 T \gg 1$)

$$\beta_{nk} = -\epsilon\omega_1 T v_{k-,n-}^+ \delta_{k,\gamma-n} e^{-i\omega_k T}. \tag{4.4}$$

Using (2.6) and (3.11), finally we have

$$|\beta_{nk}|^2 = \frac{1}{4} n k (\epsilon\omega_1 T)^2 \delta_{k,\gamma-n}. \tag{4.5}$$

Therefore the total number of photons created in the k th mode from the empty cavity is

$$N_k = \sum_{n=1}^{\infty} |\beta_{nk}|^2 = \begin{cases} \frac{1}{4}(\gamma - k)k(\epsilon\omega_1 T)^2 & k < \gamma \\ 0, & \text{otherwise.} \end{cases} \tag{4.6}$$

This result is a generalization of Ref. [8] in the short time limit ($\epsilon\omega_1 T \ll 1$) and it agrees with that result for $\gamma = 2$ and $k = 1$. It should also be noted that the maximal number of photons are created at the mode frequency

$$k = \frac{\gamma}{2} \text{ or } \omega_k = \frac{\Omega}{2}. \tag{4.7}$$

for $\gamma = \text{even}$ and at its nearest neighbor frequencies $k = (\gamma \pm 1)/2$ for $\gamma = \text{odd}$.

V. DISCUSSION

We changed the second order coupled differential equation (2.5) to the first order differential equation (3.6) by introducing the new variables (3.4). This makes it easy to deal with the differential equation and to find the perturbation series by virtue of diagonalization of $V^{(0)}$. The ϵ^1 -order solution is found explicitly and it includes the terms proportional to time and this term is relatively large compared to other terms which include only oscillating parts. Considering only those dominant terms, we calculated the number of photons created after stopping of the wall vibration. The results show that the effect of parametric resonance is

the largest at the half of the frequency of the vibrating wall ($\omega_k = \Omega/2$). This can be understood by considering the Mathieu equation (1.1) where the parametric resonance takes place most strongly for $\Omega = 2\mu$. While, in the case $\gamma > 2$, we see the other resonance effects in addition to $\omega_k = \Omega/2$, which is due to the effect of couplings with other mode frequencies in the cavity. This can be interpreted as the parametric resonance for the coupled differential equation.

Following the discussions of Ref. [8], we estimate the rate of photon generation in the several modes. Our results (4.6) which are valid only in the limit $\epsilon\omega_1 T \ll 1$ may be used to estimate the relative photon numbers depending on the mode frequency. For a long time, we assume that the photon numbers are proportional to time [5,7] or to exponential function of time [4]. Then we have a photon *distribution* proportional to Eq. (4.6). For example, taking $\gamma = 4$ we have $N_2 = 4N_D$ and $N_1 = N_3 = 3N_D$ where N_k is the number of created photons with ω_k . Here we used the same experimental parameters as considered in Ref. [8]: $\epsilon_{\max} \sim v_s \delta_{\max} / \gamma \pi c \sim 1 \times 10^{-8}$, $v_s \delta_{\max} \sim 50$ m/s, $\omega_1 / 2\pi \sim 10$ GHz for $L_0 \sim 2$ cm, and $N_D \sim 300$ photons during $T = 1$ s. For these experimental parameters, we expect that $N_2 = 1200$ photons and $N_1 = N_3 = 900$ photons.

The higher order calculations can be performed by taking the differential equation to higher order in ϵ and iterating the integral. Here we note that when we make the ϵ^p -order differential equation like (3.6) from (2.5) we consider the higher order coming from (3.2). However if we limit the problem to the dominant terms which result from the parametric resonance, the problem is simple. Let us explain these by considering the ϵ^2 -order. The differential equation can be written, in the ϵ^2 -order, as

$$\frac{d}{dt} \vec{X}_n^{(2)} = V^{(2)} \vec{X}_n^{(0)} + V^{(1)} \vec{X}_n^{(1)} + V^{(0)} \vec{X}_n^{(2)}. \quad (5.1)$$

By integrating this differential equation it is clear that the $V^{(2)}$ term gives at most the term proportional to $\epsilon^2 \omega_1 T$. This is very small compared to $(\epsilon \omega_1 T)^2$ which comes from integrating the resonance term of $X^{(1)}$. Therefore it is enough to write

$$\frac{d}{dt} \vec{X}_n^{(p)} = V^{(1)} \vec{X}_n^{(p-1)} + V^{(0)} \vec{X}_n^{(p)}, \quad (5.2)$$

to consider p -th order calculation. By iterations, we have

$$\begin{aligned}
X_{n,k\sigma}^{(p)}(t) \approx & \frac{1}{p!}(\omega_1 t)^p e^{\sigma i k \omega_1 t} \times \\
& \sum_{\sigma_{p-1}, \dots, \sigma_1, s_p, \dots, s_1} \delta_{0, -\sigma k + \Sigma_p \gamma - n} \times \\
& v_{k, \sigma; \sigma_{p-1}(\Sigma_{p-1} \gamma - n), \sigma_{p-1}}^{s_p} \times \\
& v_{\sigma_{p-1}(\Sigma_{p-1} \gamma - n), \sigma_{p-1}; \sigma_{p-2}(\Sigma_{p-2} \gamma - n), \sigma_{p-2}}^{s_{p-1}} \times \\
& \dots \times \\
& v_{\sigma_2(\Sigma_2 \gamma - n), \sigma_2; \sigma_1(\Sigma_1 \gamma - n), \sigma_1}^{s_2} \times \\
& v_{\sigma_1(\Sigma_1 \gamma - n), \sigma_1; n, -}^{s_1}
\end{aligned} \tag{5.3}$$

where $\Sigma_k = s_1 + \dots + s_k$. For large t , we should be careful about the convergency of the series. We expect that the convergency property may be provided by the factor $1/p!$. However the explicit calculations are too difficult to find the simple general form of the solution and this will be our concern of future work [13].

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